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The ϕ-ω Mixing Angle

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Under the assumption of vector-meson dominance for the decays $\pi^0 \to 2\gamma$, $\omega \to 3\pi$, and $\phi \to \rho+\pi$, we obtain the ϕ-ω mixing angle from the observed decay rates for these processes. The mixing angles thus obtained are smaller than the Okubo-Sakurai value derived from the Gell-Mann–Okubo mass formula for the vector-meson octet. The lower mixing angles obtained here correspond to only a small violation (less than 10%) of the mass-square formula. Further possible checks on the mixing angles obtained here, as well as on the hypothesis of ϕ-ω mixing and the vector-meson dominant model are discussed.

UNDER the assumption that the observed ϕ and ω are linear superpositions of ω_1 (a unitary singlet) and ω_8 (the $I=Y=0$ member of unitary octet), the corresponding mixing angle θ plays an important role in many physical calculations. Okubo[1] and Sakurai[2] calculated this mixing angle by using the mass of ω_8 as given by the Gell-Mann–Okubo mass formula, based on lowest order perturbation due to the mass splitting term. In so far as "small" violations of this mass formula need not be surprising and that the mixing angle is rather sensitive to this violation, it is pertinent to obtain this mixing angle from other independent considerations. In this note we wish to point out that if the assumption of vector-meson dominance for decay processes such as $\pi^0 \to 2\gamma$, $\omega \to 3\pi$, $\phi \to \rho+\pi$, etc., is not badly violated, then the observed decay rates for such processes indicate that the mixing angle may be smaller than the Okubo-Sakurai value. Further possible checks on the results obtained here as well as on the hypotheses of ϕ-ω mixing and vector-meson dominant model are discussed.

In the notation of Glashow,[3] the SU (3)-invariant vector-vector-pseudoscalar meson coupling has the form

$$\begin{aligned} L_{\mathrm{Int}} = \pi^0\rho^0\{&\omega(g\cos\theta + f\sin\theta) + \phi(f\cos\theta - g\sin\theta)\} \\ &+\eta^0\{\omega^2(g\cos\theta - \tfrac{1}{2}f\sin^2\theta) - \phi^2(g\sin\theta\cos\theta \\ &+\tfrac{1}{2}f\cos^2\theta) + \omega\phi(g\cos 2\theta - \tfrac{1}{2}f\sin 2\theta) + \tfrac{1}{2}f\rho^0\rho^0\} \\ &+\cdots, \end{aligned} \tag{1}$$

where g and f are the singlet and the octet (D-type) coupling constants, respectively. The space-time indices have been omitted on the right-hand side of Eq. (1). The physical ϕ and ω are related to ω_1 and ω_8 by

$$\begin{aligned} |\phi\rangle &= \cos\theta|\omega_8\rangle - \sin\theta|\omega_1\rangle, \\ |\omega\rangle &= \sin\theta|\omega_8\rangle + \cos\theta|\omega_1\rangle. \end{aligned} \tag{2}$$

θ is the ϕ-ω mixing angle, which we will choose by definition to be positive lying within the first quadrant.

Let us first consider the $\pi^0 \to 2\gamma$ decay. In the vector-meson dominant model,[4] the π^0 meson goes to a pair of neutral vector mesons $(\rho+\omega)$ or $(\phi+\omega)$, each of which then transforms into one of the final-state photons. In the limit of unitary symmetry the photon vertices satisfy the relations $G_{\omega_1\gamma}=0$, $G_{\rho\gamma}=\sqrt{3}G_{\omega_8\gamma}$. Using the pion electric-form factor to evaluate $G_{\rho\gamma}$,[5] we obtain for the decay rate,

$$\begin{aligned} P(\pi^0 \to 2\gamma) \approx \frac{\alpha^2}{192\pi}\left(\frac{m_\rho}{m_\omega}\right)^4 \{&f(1-\epsilon\cos^2\theta) \\ &+\epsilon g\sin\theta\cos\theta\}^2 m_\pi^3, \end{aligned} \tag{3}$$

where

$$\epsilon = (m_\phi^2 - m_\omega^2)/m_\phi^2, \quad \alpha = 1/137.$$

Let us next consider the $\omega \to 3\pi$ decay. If this is dominated by the $(\rho+\pi)$-intermediate state, the decay rate is given by

$$\begin{aligned} P(\omega \to \pi^+ + \pi^- + \pi^0) \simeq &(0.025)(g_{\rho\pi\pi}^2/4\pi) \\ &\times (f\sin\theta + g\cos\theta)^2 m_\pi^3, \end{aligned} \tag{4}$$

where $(g_{\rho\pi\pi}^2/4\pi)$ is estimated to be 0.5.[5]

From (1), the decay rate of $\phi \to \rho^0+\pi^0$ is given by

$$\begin{aligned} &P(\phi \to \rho^0 + \pi^0) \\ &= \frac{1}{96\pi}\left[\frac{\{(m_\phi+m_\rho)^2 - m_\pi^2\}\{(m_\phi - m_\rho)^2 - m_\pi^2\}}{m_\phi^2 m_\pi^2}\right]^{3/2} \\ &\qquad \times (f\cos\theta - g\sin\theta)^2 m_\pi^3, \end{aligned} \tag{5}$$

we then have three equations

$$\begin{aligned} [f(1-\epsilon\cos^2\theta) + g\epsilon\sin\theta\cos\theta]^2 &= a^2, \\ (f\sin\theta + g\cos\theta)^2 &= b^2, \\ (f\cos\theta - g\sin\theta)^2 &= c^2, \end{aligned} \tag{6}$$

where a^2, b^2, and c^2 are related to the $\pi^0 \to 2\gamma$-, $\omega \to 3\pi$-, and $\phi \to \rho^0+\pi^0$-decay rates through Eqs. (3), (4), and

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1 S. Okubo, Phys. Letters **5**, 165 (1963).

2 J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

3 S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963).

4 This model without the picture of ϕ-ω mixing has been applied to the various decay modes considered in the present paper by many authors. See, M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 26 (1962); S. Hori, S. Oneda, S. Schiba, and H. Hiraki, Phys. Letters **1**, 81 (1962).

5 We assume the electric-form factor of the pion is dominated by the ρ-contribution, which gives $em_\rho^2 \approx 2g_{\rho\pi\pi}G_{\rho\gamma}$. The $\rho\pi\pi$-coupling constant is estimated from the width of $\rho^0 \to \pi^+ + \pi^-$ decay ($\approx$100 MeV), which gives $g_{\rho\pi\pi}^2/4\pi \approx 0.5$.

TABLE I. Solutions for the ϕ–ω-mixing angle, the corresponding values for coupling constants, branching ratios, and decay rates, that are of interest.[a]

	$\theta=\theta_1\simeq 23°$		$\theta=\theta_2\simeq 17°$		$\theta=38°$	
	Solution I	Solution II	Solution I	Solution II	Solution I	Solution II
f (in m_π^{-1})	1.0	0.67	0.82	0.44	1.5	1.2
g/f	1.9	3.14	2.5	4.9	1.06	1.55
$P(\pi^0\to 2\gamma)$ (in eV)	6	6	6	6	21	14.5
$P(\phi\to\eta+\gamma)/P(\omega\to\pi+\gamma)$	0.46	0.32	0.30	0.17	0.75	0.69
$P(\omega\to\eta+\gamma)/P(\omega\to\pi+\gamma)$	2×10^{-2}	2.8×10^{-2}	3.0×10^{-2}	4.0×10^{-2}	1.1×10^{-3}	5.3×10^{-3}
$P(\eta\to 2\gamma)/P(\eta\to\pi^++\pi^-+\gamma)$	7.1	8.0	7.2	8.6	6.4	7.0
$P(\eta\to 2\gamma)/P(\pi^0\to 2\gamma)$	68.5	53.5	73.6	52.2	58.5	52.2
$P(\phi\to K+\bar{K})$ (in MeV)	2.6		2.9		2.0	
$P(\omega\to\pi+\gamma)/P(\omega\to\pi^++\pi^-+\pi^0)$	0.167		0.167		0.167	
$P(\rho^0\to\pi^0+\gamma)/P(\rho^0\to\pi^++\pi^-)$	$(2\sim4)\times10^{-4}$		$(2\sim4)\times10^{-4}$		$(2\sim4)\times10^{-4}$	

[a] The numbers in this table are based on the assumption of vector-meson dominant model and unitary symmetric coupling constants (apart from ϕ-ω mixing in the states). The values $\theta=\theta_1\approx23°$ or $\theta=\theta_2\approx17°$, together with the corresponding values of f and g/f are obtained by using $P(\omega\to\pi^++\pi^-+\pi^0)\approx8.4$ MeV, $P(\phi\to\rho^0+\pi^0)\approx0.4$ MeV, and $P(\pi^0\to2\gamma)\approx6$ eV [see Eq. (7)]. The value $\theta=38°$ is obtained by using the Gell-Mann–Okubo mass formula; the corresponding values for f and (g/f) are chosen to fit the widths of $\omega\to\pi^++\pi^-+\pi^0$ and $\phi\to\rho^0+\pi^0$ decays only. Solutions I and II under each column correspond to the relative sign of b and c [see Eq. (6)] being positive and negative, respectively. The quantities in the last two rows are essentially predictions of the vector-meson dominant model and do not depend upon the ϕ-ω mixing. The width of $\phi\to K+\bar{K}$ decay essentially depends upon the picture of ϕ-ω mixing, but not on the vector-meson dominant model.

(5), respectively. Equation (6) yields

$$\begin{aligned}\sin\theta&=\frac{ab\pm c(1-\epsilon)[b^2+c^2(1-\epsilon)^2-a^2]^{1/2}}{b^2+c^2(1-\epsilon)^2},\\ g&=b\cos\theta-c\sin\theta,\\ f&=b\sin\theta+c\cos\theta.\end{aligned}\tag{7}$$

The experimental values for the decay rates are given by

$$\begin{aligned}&P(\pi^0\to2\gamma)\simeq3\sim6\text{ eV},^{6}\\ &P(\omega\to\pi^++\pi^-+\pi^0)\approx8.4\text{ MeV},^{7}\\ &P(\phi\to\rho^0+\pi^0)<0.4\text{ MeV}.^{8}\end{aligned}\tag{8}$$

Using (7) and (8) we obtain two possible solutions for θ, which are

$$\theta_1\simeq23°\quad\text{and}\quad\theta_2\simeq17°.\tag{9}$$

In obtaining the above solutions we have used the upper limits for $\pi^0\to2\gamma$- and $\phi\to\rho^0+\pi^0$-decay widths. Any smaller value for either (or both) of them will lead to even smaller mixing angles than obtained here,[9] although, qualitatively, the results remain unchanged. Corresponding to each of the above solutions for the mixing angle, there are two possible volues for f and g/f, which arise since the relative sign of b and c can be either positive or negative.[10] These values are given in Table I and the two possible solutions for each angle are referred to as solutions I and II, respectively.

Thus we see that the assumption of vector-meson dominant model and the observed decay rates [Eq. (8)] are not consistent with the exact validity of the Gell-Mann–Okubo mass formula, which yields $\theta\simeq38°$. Let us first note that the mixing angles obtained here (θ_1 or θ_2) correspond to a mass of ω_8 nearly equal to 965 or 978 MeV. This is only 3–4% higher than the value predicted by the mass formula (930 MeV). Such small violations from the mass formula are not unexpected. On the other hand, if we accept the mixing angle $\theta\simeq38°$, then we can obtain f and g/f from the $\omega\to\pi^++\pi^-+\pi^0$ and $\phi\to\rho^0+\pi^0$ decay rates. The values thus obtained determine the $\pi^0\to2\gamma$-decay rate (in the vector-meson dominant model). These have also been listed in Table I. It may be noticed that these values are bigger than the experimental π^0 width by a factor[11] of $3.5\sim7$ (soln I) or $2.5\sim5$ (soln II).

We next enquire what are the other possible sources of obtaining the mixing angle θ and the coupling constants f and g. One way will be to look at some of the other decay modes of ϕ and ω mesons which (in the vector-meson dominant model) involve these parameters in a combination different from those occurring in Eq. (6). Two such combinations are

$$\begin{aligned}M(\phi\to\eta+\gamma)&\propto-(g\sin\theta+f\cos\theta)\\ M(\omega\to\eta+\gamma)&\propto(g\cos\theta-f\sin\theta).\end{aligned}\tag{10}$$

[6] R. G. Glasser, N. Seeman, and B. Stiller, Phys. Rev. 123, 1014 (1961). G. Von Dardel *et al.*, Phys. Letters 4, 51 (1963).

[7] N. Gelfand, D. Miller, M. Nussbaum, J. Ratau, J. Schultz, *et al.*, Phys. Rev. Letters 11, 436 (1963). Earlier references are cited here.

[8] P. L. Connolly *et al.*, Brookhaven National Laboratory and Syracuse University (unpublished). Earlier references are cited here. We are indebted to Professor J. Leitner for many communications regarding the ϕ meson.

[9] For example, in the limit $c^2\to0$ [i.e., $P(\phi\to\rho^0+\pi^0)\to0$], there is only one solution $\theta\approx20°$, while in the limit $c^2\to0$ and $P(\pi^0\to2\gamma)\approx3$ eV, θ reduces to 15°.

[10] Note that an over-all change in the sign of both f and g is irrelevant for our purpose and hence not considered. So only the relative sign of b and c matters.

[11] The uncertainty by a factor of two arises due to the uncertainty in the experimental value for π^0 width (see Ref. 6).

In Table I, we list the values of these decay rates compared to that of $\omega \to \pi+\gamma$, whose decay amplitude is proportional to $\sqrt{3}(g\cos\theta+f\sin\theta)$. We may note from the table that a determination of either of the ratios $P(\phi\to\eta+\gamma)/P(\omega\to\pi+\gamma)$ or $P(\omega\to\eta+\gamma)/P(\omega\to\pi+\gamma)$ (especially of the latter) could serve to distinguish at least between the solutions corresponding to $\theta\approx 38°$ and $\theta=\theta_1$ or θ_2 obtained here. Since $\omega\to\pi+\gamma$-decay rate is more or less known, it will be extremely interesting to have experimental information on $\phi\to\eta+\gamma$ and $\omega\to\eta+\gamma$ decays.

Within the vector-meson dominant model, the ϕ-ω mixing again plays an important role in η decays. The branching ratio $P(\eta\to 2\gamma)/P(\eta\to\pi^++\pi^-+\gamma)$ is given by

$$\frac{P(\eta\to 2\gamma)}{P(\eta\to\pi^++\pi^-+\gamma)}\simeq 7.4(1-\beta)^2\,, \tag{11}$$

where

$$\beta=\frac{1}{3}\left(\frac{m_\rho}{m}\right)^4\{1-2\lambda(g/f\sin2\theta+\cos2\theta)+\lambda^2\cos2\theta(2g/f\sin2\theta+\cos2\theta)\}\,, \tag{12}$$

$$\frac{2}{m^2}=\frac{1}{m_\omega^2}+\frac{1}{m_\phi^2}\,,\quad \frac{2\lambda}{m^2}=\frac{1}{m_\omega^2}-\frac{1}{m_\phi^2}\,.$$

The values of the above branching ratio for the different solutions are listed in Table I. We note that this ratio is nearly $7\sim 8$ for all the different solutions and hence cannot distinguish between them. However, an interesting fact is that without ϕ-ω mixing[4] ($\omega=\omega_8$), $\theta\to 90°$ and the above ratio reduces to nearly 3.8. Thus a determination[12] of this branching ratio may serve to check the basic hypothesis of ϕ-ω mixing. We further note, in the limit of strict unitary symmetry (hence no ϕ-ω mixing), one obtains (in any model)

$$M(\pi^0\to 2\gamma)=\sqrt{3}M(\eta\to 2\gamma)$$

which yields

$$P(\eta\to 2\gamma)/P(\pi^0\to 2\gamma)=\tfrac{1}{3}(m_\eta/m_\pi)^3\simeq 20\,.$$

If we assume that violation of unitary symmetry is represented by an effective ϕ-ω mixing interaction and that the intermediate vector meson vertices (in the vector-meson dominant model) are, otherwise, governed by unitary symmetric coupling constants, then ϕ-ω mixing alters the above ratio to nearly 60 (which is nearly constant for all the different solutions, as can be seen from Table I). A determination of this ratio will, therefore, be of interest.

Since our discussion is heavily based upon the vector-meson dominant model, it is in order to remark about the possible test of validity of this model in various processes independently of the picture of ϕ-ω mixing. We first note that in the vector-meson dominant model, both $\omega\to\pi+\gamma$ and $\omega\to\pi^++\pi^-+\pi^0$ decays are dominated by $(\rho+\pi)$-intermediate state and hence the ratio of their rates depends only[13] on the $\rho-\gamma$ vertex. If we estimate the vertex as before,[5] the branching ratio $P(\omega\to\pi+\gamma)/P(\omega\to\pi^++\pi^-+\pi^0)$ is predicted to be 0.167.[4] The experimental branching ratio $P(\omega\to \text{neutrals})/P(\omega\to\pi^++\pi^-+\pi^0 \text{ or } \gamma)$ is reported to be 0.17 ± 0.04,[14] 0.11 ± 0.02.[15] It seems that the decays $\omega\to\pi+\gamma$ and $\omega\to\pi^++\pi^-+\pi^0$- are, respectively, the dominant modes in the numerator and denominator processes. Thus the vector-meson dominant model seems to be consistent in this case. We further note that the relatively large width of $\omega\to\pi^++\pi^-+\pi^0$ decay implies a relatively large value for $\omega\rho\pi$ coupling in the vector-meson dominant model, which, together with the small width of $\phi\to\rho^0+\pi^0$-decay implies $g_{\phi\rho\pi}/g_{\omega\rho\pi}\ll 1$. This is consistent with the conclusion obtained from production experiments $\pi+N\to\pi+N+\phi(\omega)$.[16] Another branching ratio, that will be of interest is $P(\rho^0\to\pi^0+\gamma)/P(\rho^0\to\pi^++\pi^-)$. In the vector-meson dominant model, the width of $\pi^0\to 2\gamma$-decay determines that of the $\rho^0\to\pi^0+\gamma$ decay and the said branching ratio is predicted to be $(2\sim4)\times10^{-4}$.[11] At the moment there is no reliable information on this branching ratio.

It seems that the best way to determine the mixing angle θ independently of the vector-meson dominant model is an accurate determination of the width of $\phi\to K+\bar{K}$ decay. In the limit of unitary symmetry, the relevant coupling constant is determined in terms of the width of $\rho\to 2\pi$ decay, and the width of $\phi\to K+\bar{K}$ decay is then predicted to be $\cos^2\theta$ (3.2 MeV). The best experimental value for the full width seems to be[8]

$$\Gamma(\phi)_{\text{total}}=3.1\pm0.8\text{ MeV}.$$

The partial width for $\phi\to K+\bar{K}$ seems to be nearly equal to its full width. Thus, although the above results favor the lower mixing angles, obtained here, the errors are large enough not to permit any definite conclusion.

We finally turn to production experiments, which could provide information on the mixing angle θ. One way, as suggested by Glashow,[3] is to examine the reactions $K^-+P\to\Lambda+\phi(\omega)$. Unfortunately, it is hard to draw any useful information from these reactions, until one can separate out the K and K^* exchange

[12] See F. C. Crawford, R. A. Grossman, L. J. Lloyd, L. R. Price, and E. C. Fowler, Phys. Rev. Letters **11**, 564 (1963). The best value for $x=P(\eta\to 2\gamma)/[P(\eta\to +-0)+P(\eta\to +-\gamma)]$ as quoted in footnote 9 of this reference is 1.42 ± 0.36, also $P(\eta\to\text{charged})/P(\eta\to +-0)=1.26\pm0.08$. Combining the two results, it seems that the best value for $P(\eta\to 2\gamma)/P(\eta\to\pi^++\pi^-+\gamma)=6.85_{-2.67}^{+4.85}$, which is consistent with results obtained here (see Table I).

[13] Since $\rho\pi\pi$ vertex is known from the observed ρ width.

[14] R. Armenteros *et al.*, Proceedings of the Sienna International Conference on Elementary Particles, 1963 (to be published).

[15] B. Buschbeck-Czapp *et al.*, Proceeding of the Sienna International Conference on Elementary Particles, 1963 (to be published); J. J. Murray *et al.*, Phys. Letters **7**, 358 (1963); and Ref. 7.

[16] Y. Y. Lee, N. D. C. Moebs, B. P. Rose, D. Sinclair, and J. C. Vander Velde, Phys. Rev. Letters **11**, 508 (1963).

contributions.[17] Another interesting set of reactions involving ϕ and ω-productions are $\pi^+ + P \to N^{*++} + \phi(\omega)$. Recently, Meshkov *et al.*[18] analyzed these reactions together with some of the partner processes in unitary symmetry. Assuming that the SU (3) matrix elements are not perturbed by the mass splitting interactions, they derived the cross-section sum rule

$$F_a\sigma_a = F_b\sigma_b + 3F_c\sigma_c - 3F_d\sigma_d\,, \tag{13}$$

where σ_a, σ_b, σ_c, and σ_d are the cross sections for the reactions $K^+ + P \to N^{*++} + K^{*0}$, $\pi^+ + P \to N^{*++} + \rho^0$, $\pi^+ + P \to N^{*++} + \omega_8$, and $\pi^+ + P \to Y_1^{*+} + K^{*+}$, respectively. F_a, F_b, F_c, and F_d are the respective kinematic factors. Experimentally the ratio $\sigma(\pi^+ + P \to N^{*++} + \phi)/\sigma(\pi^+ + P \to N^{*++} + \omega)$ is much less than unity, which leads to

$$\sin^2\theta \simeq [F_a\sigma_a + 3F_d\sigma_d - F_b\sigma_b]/3F_f\sigma_f\,,$$

where σ_f and F_f are, respectively, the cross section and kinematical factor for the reaction $\pi^+ + P \to N^{*++} + \omega$. For what it is worth, the best estimate of θ, from the data presented by Meshkov *et al.*[18] turns out to be

$$\theta \simeq 23° \sim 27°$$

which is close to the solution θ_1 obtained here.

To summarize, we have attempted to obtain the ϕ-ω-mixing angle θ independently of the Gell-Mann–Okubo mass formula for the vector-meson octet. The considerations presented here indicate mixing angles smaller than the Okubo-Sakurai value, based on the use of the mass formula. The lower mixing angles obtained here correspond to only a small violation (less than 10%) of the mass-square formula. Experimental information on the various decay rates and branching ratios, that we feel, will be of great interest in testing some of the ideas and results presented here are listed in Table I.

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[17] There seems to be good indications from Ref. 8 that both K and K^* exchange contribute to these reactions.

[18] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

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Reciprocal Bootstrap Relationship of the Octet Baryon and the Decuplet Baryon*

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The reciprocal bootstrap relationship of the octet of $(\frac{1}{2})^+$ baryons and the decuplet of $(\frac{3}{2})^+$ baryons is studied making use of the static approximation. If we regard the octet baryon as a $B_8\Pi_8$ bound state due to octet and decuplet baryon exchange, we obtain $\gamma_{10} \approx 7d^2$ and $d/f \approx 2.2$, where γ_{10} is the $\bar{B}_{10}B_8\Pi_8$ coupling constant and d and f are the d and f coupling constants of $\bar{B}_8B_8\Pi_8$ coupling. If octet vector meson exchange processes are included and if we assume the vector theory (gauge theory) of strong interactions, we obtain $\gamma_{10} < 7d^2$ and $d/f < 2.2$. If we regard the decuplet baryon as a $B_8\Pi_8$ bound state due to octet baryon exchange, we obtain $\gamma_{10} \approx 4d^2$ for the ratio $d/f = 2.2$.

ABOUT two years ago, Chew proposed the idea of a reciprocal bootstrap relationship between the nucleon and the (3,3) resonance.[1] Since that time, the accumulated experimental data, especially the recent discovery of the Ω^- particle,[2] seem to have proved the validity of the first-order broken eightfold way.[3–5] Here we would like to check the idea of a reciprocal bootstrap relationship between the octet of $(\frac{1}{2})^+$ baryons and the decuplet of $(\frac{3}{2})^+$ baryons, making use of the static approximation.[6] This is interesting since the bootstrap

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[1] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).

[2] V. E. Barnes, P. L. Connolly, D. J. Crennell, B. B. Culwick, *et al.*, Phys. Rev. Letters **12**, 204 (1964).

[3] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report CTSL-20, 1961 (unpublished).

[4] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[5] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 947 (1962).

[6] This problem has been discussed by V. Singh (to be published) making use of the static model, but he has made mistakes in the treatment of the coupled N/D method. This problem has also been discussed by I. S. Gerstein and K. T. Mahanthappa, but their result is contradictory with a Chew-type coupled N/D method. I. S. Gerstein and K. T. Mahanthappa, Nuovo Cimento **32**, 239 (1964).